Why Knowledge Should Not Be Typed: An Argument against the Type Solution to the Knowability Paradox

by

MASSIMILIANO CARRARA
Department of Philosophy, University of Padua

and

DAVIDE FASSIO
Department of Philosophy, University of Geneva

Abstract: The Knowability Paradox is a logical argument to the effect that, if there are truths not actually known, then there are unknowable truths. Recently, Alexander Paseau and Bernard Linsky have independently suggested a possible way to counter this argument by typing knowledge. In this article, we argue against their proposal that if one abstracts from other possible independent considerations supporting reasons for typing knowledge and considers the motivation for a type-theoretic approach with respect to the Knowability Paradox alone, there is no substantive philosophical motivation to type knowledge, except that of solving the paradox. Every attempt to independently justify the typing of knowledge is doomed to failure.

Keywords: Knowability Paradox, type theory, epistemic paradoxes

1. Introduction

A LOGICAL ARGUMENT known as the Knowability Paradox starts from the assumption that every truth is knowable and leads to the consequence that every truth is also actually known. Given the ordinary fact that some true propositions are not actually known, the argument then concludes, by modus tollens, that there are unknowable truths.

The straightforward conclusion of the argument seems to be seriously problematic for those assuming that every truth is, at least in principle, knowable. For instance, it is problematic for some semantic anti-realist theories that require an epistemic characterization of the notion of truth and certain “optimistic” views in epistemology and philosophy of science, according to which no truths exceed our cognitive capacities to grasp them.\(^1\)

Many strategies have been suggested in order to avoid the paradoxical conclusion.\(^2\) Recently, Alexander Paseau (2008) and Bernard Linsky (2009) have inde-

\(^1\) An example of this view is the so-called “Gödelian Optimism”.
\(^2\) For an introduction and a general overview of the various critical approaches to the paradox, see Brogaard and Salerno (2009) and Kvanvig (2006).
pendently suggested a possible way to counter the argument by typing knowledge.\(^3\)
The aim of the present article is to point out a criticism of such a solution strategy to the Knowability Paradox.\(^4\) We argue that, if one abstracts from other possible independent considerations supporting reasons for typing knowledge and considers the motivation for a type approach with respect to the Knowability Paradox alone, there is no substantive philosophical motivation for introducing knowledge types except that of solving the paradox. The introduction of types is not independently justified by any property actually held by knowledge, and is therefore \textit{ad hoc}.\(^5\)

2. The Knowability Paradox

The Knowability Paradox is a logical argument having the following structure.\(^6\) Take a common system of classical modal propositional logic plus the epistemic operator \(K\), where “\(Kp\)” stands for “someone, at some times, knows that \(p\)”, and “\(p\)” is a proposition. Assume as uncontroversial the validity of two properties of knowledge: 1) the distributivity over conjunction (if a conjunction is known, then its conjuncts are also known), and 2) the factivity of knowledge (if a proposition is known, then it is true). Formally:

\[
\begin{align*}
\text{(Dist)} \quad & K(p \land q) \vdash Kp \land Kq \\
\text{(Fact)} \quad & Kp \vdash p
\end{align*}
\]

The argument makes use of two uncontroversial modal rules: the rule of necessitation and the rule of interdefinability of necessity and possibility. Formally:

\[^3\text{Notice, however, that this solution strategy is not a complete novelty: in the anonymous referee report to Frederic Fitch’s article, where the paradox first appeared, Alonso Church suggested the possible use of a type-distinction in the argument. See Church (2009). However, only recently has this approach to the problem been seriously taken into account. Notice also that Church’s paper was not published when Linsky and Paseau formulated their own positions on the issue.}

\[^4\text{The type strategy has recently been the target of some other criticisms. On one side, it has been remarked that a hierarchical approach to the paradox would meet obstacles in the characterization of knowledge type-levels. On the other side, examples of propositions have been shown against which the type approach would be ineffective. For the former criticism, see Williamson (2000, p. 281); for the latter, see Williamson (2000, pp. 281–282) and Hart (2009, pp. 322–323). Williamson’s arguments partially foreshadow some of the criticisms advanced in section 4. Another interesting criticism of the type solution has been raised by Florio and Murzi (2009): the authors gave an argument (called Paradox of Idealization) structurally similar to the Knowability Paradox, but resistant to hierarchical approaches.}

\[^5\text{In this article, by the expression “\textit{ad hoc}” we do not mean any technical use of the term. Rather, the term is here equivalent to “deprived of a motivation independent from that of solving the specific problem at issue”.}

\[^6\text{The argument first appeared in 1963 in an article by F. Fitch, entitled “A Logical Analysis of Some Value Concepts”.}

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(Nec) if $\vdash p$, then $\vdash \Box p$
(ER) $\Box \neg p \vdash \neg \Diamond p$.

Now, assume the Knowability Principle, according to which every true proposition is knowable. Formally:

(KP) $\forall q (q \rightarrow \Diamond Kq)$

Assume also that there is at least one truth that is not known (by anyone, at any time):

(NO) $\exists r (r \land \neg Kr)$

Given these assumptions, the argument runs as follows:

2) $p \land \neg Kp$ instantiation of (NO)
3) $(p \land \neg Kp) \rightarrow \Diamond K(p \land \neg Kp)$ by the substitution of $q$ in (KP) with (2)
4) $\Diamond K(p \land \neg Kp)$ by (2) and (3)
5) $K(p \land \neg Kp)$ assumption “per absurdum”
6) $Kp \land K\neg Kp$ by (5) and (Dist)
7) $Kp \land \neg Kp$ applying (Fact) to (6)
8) $\neg K(p \land \neg Kp)$ by (5)–(7), from the inconsistency of (7)
9) $\Box \neg K(p \land \neg Kp)$ by (8) and (Nec)
10) $\neg \Diamond K(p \land \neg Kp)$ by (9) and (ER)

(10) is inconsistent with (4). If so, (NO) and (KP) are incompatible and one of them must be abandoned. One could deny (NO), obtaining the conclusion that there are no unknown truths, i.e., every truth is known. Otherwise, one could deny (KP), obtaining that not every truth is knowable, i.e., there are unknowable truths. The conclusion of the argument is thus that, if there are unknown truths, there are unknowable truths. It seems an evident and ordinary fact that we are not omniscient beings and that some truths are not actually known. Given this assumption, the argument arrives at the conclusion that there are unknowable truths.\(^7\)

3. A Typed Solution of the Paradox

A proposal for resolving the paradox based on typing knowledge has been recently developed by Alexander Paseau and Bernard Linsky. The account is

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\(^7\) Notice that the conclusion of the paradox is problematic only for views which want to maintain that all truths are knowable and reject that there are no unknown truths. The argument need not be paradoxical for views not committed to those theses. See, for example, Williamson (2000).
based on the following two basic rules (where \( \varphi \) is a variable ranging over any proposition):\(^8\)

1. If \( \varphi \) has no occurrences of \( K \), \( \varphi \) is of type 0 (\( \varphi_0 \))
2. If \( \varphi \) is of type \( n \), then \( K\varphi \) is of type \( n + 1 \) (\( \varphi_{n+1} \))

A further rule must be added for the typing of complex propositions:

3. If \( \psi \) is a complex proposition, and the proposition included in \( \psi \) with the maximum type is of type \( n \), then \( \psi \) is of type \( n \).\(^9\)

According to Paseau and Linsky, given the above characterization of types of knowledge, the Knowability Paradox is solved. In fact, let \( p \) be of type 0. Steps (5) – (7) then run as follows:

\[
\begin{align*}
5^*) & \quad K_2(p_0 \land \lnot K_1p_0) \quad \text{assumption} \\
6^*) & \quad K_2p_0 \land K_2\lnot K_1p_0 \quad \text{by (5*) and (Dist)} \\
7^*) & \quad K_2p_0 \land \lnot K_1p_0 \quad \text{applying (Fact) to (6*)}
\end{align*}
\]

Unless higher types collapse on lower ones (i.e., in steps (5*)–(7*), \( K_2 \) implies \( K_1 \)), line (7*) is not a contradiction. There is no incoherence in not knowing \( p \) at the lower level and knowing it at the higher level. Thus, if (7*) is consistent, (5*) can be maintained, and (10) \( \lnot \Diamond K(p \land \lnot Kp) \), the contradictory of (4), cannot be derived. Hence, both assumptions (KP) and (NO) can be consistently maintained.

4. How Can the Introduction of Types Be Justified?

As we said above, we think that the type-solution strategy is problematic for the following reason: without some substantive motivation independent from the mere aim of solving the paradox, the introduction of the type-distinction cannot avoid a standard criticism of other solutions to the argument, namely that of being *ad hoc*\(^{10}\). Such an introduction must be motivated by some further substantial reason distinct

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\(^8\) Notice that the accounts of the two authors diverge in some respects. Whereas Paseau’s account is more focused on the formal aspects of the approach, Linsky’s is much more focused on general philosophical aspects and it lacks a presentation of the formalism. We are not interested here in specific aspects of each account. Rather, we are interested in the philosophical motivations for the strategy in general. Thus, here we present the main lines of the typed approach in a general form, without entering into the details of each account, confident that the specificities of each approach will not affect the validity of our criticism.

\(^9\) Notice that only Paseau explicitly mentions a principle like our (3). However, the use of the principle is implicit in Linsky’s proposal.

\(^{10}\) *Ad hoc*ness is a common criticism of many other strategies for solving the paradox. See, for instance, Tennant’s restriction strategy (1997) and the criticism of *ad hoc*ness in Hand and Kvanvig (1999).
from that of avoiding the paradoxical conclusion; however, we will argue that there are no independent reasons motivating a type-distinction of knowledge.

But before we give our arguments, let us address a possible reply to the objection of *ad hoc*ness. One could argue that the introduction of types in the case of the paradox is perfectly in order even if it is *ad hoc*, and thus that it does not need any independent motivation. According to this possible reply, the type-solution would be just the application of a formal tool to a logical argument, not in need of any further justification. In our view, this answer is unsatisfactory. The *Knowability Paradox* is not merely an exercise in logical analysis deprived of any concrete application to extra-logical domains. Rather, it is meant to say something substantive about the nature of truth and the limits of knowledge. The argument is considered paradoxical in the measure to which it affects some basic intuitions concerning concepts whose analysis belongs to different extra-logical domains. In general, if the introduction of a logical tool does not reflect some real aspects of the object that that logic is supposed to represent, then its introduction cannot be considered motivated.11 For this reason, the indiscriminate typing not grounded in some actual features of knowledge is not self-justified. The need for a motivation for the type-distinction remains.12

Given the need for a further substantive independent motivation, it has been argued that such further motivation has to be found in the analogy with the effectiveness of the type-strategy for other paradoxes. The type-solution is effective not only for this paradox, but for a range of logical paradoxes, including some epistemic ones such as the *Knower Paradox* and the *Paradox of the Preface*.13 The type-theoretic approach in the case of Fitch’s argument seems to be less *ad hoc* than in other cases because it provides a solution to a family of epistemic paradoxes. Furthermore, it has been argued that typing knowledge does not differ in many relevant respects from typing truth. If it is not *ad hoc* to type truth in order to escape semantic paradoxes such as the Liar, the same should be true for knowledge.14

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11 This kind of objection is familiar in philosophical logic. For example, some axioms of logical systems such as S5 and S4 have been rejected in deontic and epistemic logics because they misrepresent the concepts whose relations they are supposed to formalize.

12 An implicit objection to the criticism of *ad hoc*ness that we did not consider in our present reply consists in arguing that, in general, *ad hoc* strategies are not problematic in themselves. According to this view, *ad hoc* solutions to philosophical problems can be inelegant and unattractive, but from the methodological point of view they are not wrong. Obviously, such a view will not consider problematic our charge of *ad hoc*ness to the paradox. However, objections of *ad hoc*ness are considered important standard criticisms to some suggested attempts at solving the paradox. See, for example, Hand and Kvanvig (1999). Those philosophers argue that an *ad hoc* solution to the paradox is not just unattractive but wrong, insofar as it is a way to escape the problem rather than to legitimately solve it. We thank an anonymous reviewer for a helpful comment on this point.


A reply to these arguments by analogy is that the generality of a solution strategy is not sufficient in itself to make principled its use. Type-theorists could answer to this reply that they have independent reasons for adopting a type-theoretic approach to epistemic paradoxes and thus to type knowledge in general. In particular, it has been argued that the introduction of knowledge types is motivated for epistemic paradoxes involving self-reference, such as the Knower Paradox.\textsuperscript{15} We do not have the space to explain here why we do not think this possible answer is convincing.\textsuperscript{16} Rather, in order to meet such a possible counter-reply by type-theorists, we are disposed to bite the bullet and weaken the thesis defended in this article. The weaker claim we will argue for is that, if one abstracts from other possible independent considerations supporting reasons for typing knowledge and considers the motivation of a type approach with respect to the Knowability Paradox alone, there is no substantive philosophical motivation for introducing knowledge types except that of solving the paradox.\textsuperscript{17}

Let us now consider our argument. Remember that, as previously said, the introduction of knowledge types for solving the Knowability Paradox can be independently motivated only if it reflects some actual distinction present in (some feature of) knowledge. Two possible strategies seem to exhaust the possible ways in which types can be grounded in real features of knowledge, depending on the nature of those features. First, distinct types of knowledge could reflect actual differences in the states of knowledge: for example, differences in situational or psychological aspects of states (let us call those strategies “state-based distinction strategies”). Differences in states of knowledge can be distinguished from differences in the content of knowledge, that is, differences due to some property of the

\textsuperscript{15} For a defence of the motivation of type-theoretic approaches for paradoxes involving self-reference, see, for example, Parsons (1974) and the subsequent literature.

\textsuperscript{16} Let us briefly address in this footnote why we do not think such a possible answer is convincing. First, we think that it is important to distinguish the use of type-theory for paradoxes involving self-reference from the use of type-theory for paradoxes that do not involve self-reference. The type-solution is usually adopted to solve problems involving self-reference, and the defences of the motivation of type-theoretic approaches concern these specific applications, but the Knowability Paradox does not rely on any sort of self-reference. On the difficulty of assimilating paradoxes not involving self-reference with those involving self-reference, see Linsky (2009, pp. 168–169) and the literature mentioned therein. Therefore, the analogy with other paradoxes for which the type approach would be motivated seems to break down. Furthermore, the introduction of knowledge types for epistemic paradoxes involving self-reference has been considered motivated precisely and exclusively because those paradoxes involve self-reference, and this weakens the thesis that the motivation for a type-theoretic approach to these paradoxes provides a reason for the indiscriminate typification of knowledge, including in cases of epistemic paradoxes not involving self-reference. There are further problems with the analogy with typing truth. The analogy, as Paseau himself admits, is not complete. There are important disanalogies between typing truth and typing knowledge, mainly due to the lack of minimality conditions on the hierarchy of types in the case of knowledge. Here, for reasons of space and because of the complexity of the matter, we cannot give an explanation of those disanalogies. See Paseau (2008, pp. 160–162).

\textsuperscript{17} Hereafter, the weakening of our target thesis will be held implicit when not explicitly mentioned.
proposition expressed by the content. According to the second possible strategy, distinct types could reflect a difference in the content of knowledge (content-based distinction strategies). We assume that every feature of knowledge that is not ascribable to the content of knowledge can be considered a feature of the state of knowledge and described as such. Consequently, there are no features of knowledge not included in those two categories on the basis of which different kinds of knowledge can be distinguished. The distinction is exhaustive. Thus, a distinction of types can be grounded either in a distinction of features of the state or of the content of knowledge. The arguments in the next part of this article are meant to show that neither of these strategies is viable. A consequence is that types cannot reflect actual features of knowledge, and thus there are no substantive ways to independently motivate the introduction of types in the case of the Knowability Paradox.

5. State-Based Distinction Strategies

Let us begin by considering state-based distinction strategies. As stated, the basic idea behind this strategy consists in finding some actual differences between kinds of knowledge states, and then linking such differences to specific type-levels. There are many ways according to which kinds of knowledge states can be distinguished. A first intuitive way is according to some difference in their psychological features. For example, we can distinguish knowledge states given different degrees of certainty. Another kind of feature of knowledge states allowing internal distinctions is constituted by situational aspects of states. For example, knowledge states can be relativized to specific subjects and times. Still another way to distinguish states is through differences in the processes of their acquisition.

In our view, this strategy is affected by two main problems. A first difficulty is that, if types are supposed to reflect kinds of knowledge states, it is not sufficient that there is a distinction amongst such states; the distinction should also reflect the way in which type-levels are actually formed and in relation between them. This means that a type-distinction based on states of knowledge requires a hierarchical structure between kinds of states analogous to the type-levels in the formalism. The problem is that knowledge types, at least according to the way in which the type-solution has been traditionally characterized, are de facto distinguished exclusively on the base of knowledge content. The content-dependence of type-levels is

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18 We consider sufficiently intuitive that, if a feature does not belong to the content of knowledge, then it can be considered a feature of the state of knowledge. However, there could be objections to this. Alternatively, state-based features could be defined as those which are not content-based features. This change does not in any way affect our arguments that follow.
a common trait of type-theory in general: every type-level is characterized as being higher than the type-level of its content, and there are no other factors different from the type-level of the content playing a role in the determination of types. For example, according to the rules of type-formation we presented in section 3, the type-level of each occurrence of K is determined by the highest type-level of other occurrences of K in the content of K itself. Therefore, according to this specific characterization of types, the only way in which an effective distinction of kinds of knowledge would reflect the distinction of types is if kinds of knowledge were distinguished on the basis of a specific aspect of their epistemic content. According to such a traditional characterization of types, differences in features of knowledge states do not play any role in the formation of types, and thus cannot reflect differences between types. Is it possible to formulate a type-theory involving an alternative definition of types based on a distinction of states? Surely, such a theory would be radically different from the traditional ones. According to such an alternative theory, types would not be defined in terms of their reciprocal relations. Even admitting the possibility of such an alternative distinction of types, it is unclear to what extent such a theory could still be considered a type-theory.

A second general problem of the state-based distinction strategy lies in the generality of the reading of K in the Knowability Paradox. K is meant to express an unrestricted state of knowledge, in the sense that it disregards any specificity of knowledge states. According to this reading, distinctions between features of knowledge states are neglected. K is taken to express so general a notion of knowledge for the following reason: consider the second assumption of the paradox, (NO); (NO) says that there are unrestrictedly unknown truths, i.e., truths which de facto are not known by anyone, at any time, in any circumstance, through any method, and so on. Now, there cannot be some kind of knowledge grasping such unrestrictedly unknown propositions, because there are no kinds of knowledge according to which such propositions are not ignored.19

If according to (NO) there are unrestrictedly unknown truths, in proposition (2) \((p \land \neg Kp)\) – which is an exemplification of (NO) – K is unrestrictedly generalized. Such generalization prevents the use of a state-based distinction of knowledge from motivating the type-distinction. In fact, a condition for the type-distinction being effective against the paradox is that in proposition \((5^*)\), \(K_2(p_0 \land \neg K_1p_0)\), the type-level of the occurrence of K outside the brackets must be higher than the

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19 Here is an example of such unrestrictedly unknown propositions from Williamson (2000, p. 272): “Either my office contains an even number of books at noon on 11 October 1999 (time t) or it does not. I could find out by counting whether it contains an even number of books at t. But I will not count them; nor will anyone else. As a matter of contingent fact, no one will ever know whether my office contains an even number of books at t. Thus either it is an unknown truth that my office contains an even number of books at t or it is an unknown truth that my office contains an odd number of books at t.” The true proposition that Williamson’s office contains an even/odd number of books at t is unrestrictedly unknown.
type-level of the occurrence of K inside the brackets, and the two type-levels must not collapse. Otherwise, an inconsistency occurs and the contradictory conclusion of the paradox follows. But if K in proposition (2) \((p \land \neg Kp)\) is meant to express unrestrictedly generalized knowledge, it is meant to be so general as to include every kind of knowledge state. Proposition (2) says that \(p\), and that there are no kinds of knowledge state by which it is known that \(p\). A consequence is that, if type-levels reflect kinds of knowledge state, then the type-level of the K-occurrence in (2) is rightly represented by a variable ranging over every type-level:²⁰ there is no type-level higher than that of K in proposition (2). Thus, in proposition (5), \(K(p \land \neg Kp)\), the type-level of the occurrence of K outside the brackets cannot be higher than the type-level of the occurrence of K inside them; an inconsistency obtains, and the paradox follows. In conclusion, if types correspond to kinds of knowledge state, then the introduction of types does not escape the paradoxical conclusion: the type-strategy would not be effective against unrestrictedly unknown propositions such as (2).²¹

6. Content-Based Distinction Strategies

We have so far rejected one of the two possible strategies exhausting the ways in which types can be grounded in real features of knowledge, namely, the state-based distinction strategy. Let now consider the second possible strategy, based on the distinction of types through a distinction of knowledge contents. By content-based

²⁰ Formally: \(\forall t (p \land \neg Ktp)\), where \(t\) stands for a type-level.

²¹ For reasons of space, we did not address a set of issues concerning specific attempts to ground types on states’ differences. Let us briefly consider some of these issues in the present footnote. A first problem affecting some approaches based on state-based distinctions is that, for type-levels reflecting kinds of states, there should be an isomorphism between the internal relations amongst type-levels and the internal relations amongst kinds of knowledge. This is problematic for all states’ distinctions whose borders are vague or relative to specific contexts. In fact, the distinction of type-levels required for the solution of the paradox must be neat. Each occurrence of knowledge should hold a specific type; otherwise, if types are not clearly distinguished, there would be a risk of a collapse of different types to the same level, whose consequence would be the rehabilitation of the paradox. Such a problem affects, for example, psychological distinctions, where borders of the distinctions are vague and often dependent upon contexts of assessment. Another set of problems is specific to a distinction of types based on a distinction of knowledge states through methods of their acquisition. First, the characterization of methods should be such that for each proposition there are at least two methods for acquiring its knowledge. In fact, if we have a proposition \(q\), such that there is only one method for acquiring knowledge of that \(q\), and we link a type (say \(K_0\)) to a state of knowledge acquired through this method, the proposition \(q \land \neg K_0q\) cannot be known. This is because we cannot know the conjunction \(q \land \neg K_0q\) either through a different method (because of the assumption that there is only this method for knowing \(q\)) or through the same method (on pain of contradiction). But it is controversial that for every sort of proposition there are two methods of acquisition of such a proposition. Second, in general, methods are not disposed hierarchically; however, it is not clear how this aspect of types is reflected by methods.
distinctions we mean those based on differences due to some property of the proposition expressed by the knowledge content. Notice first that, as said above, according to the way in which type-theories are traditionally formulated, types are defined and formed on the ground of the type-level of their content. Thus, at least apparently, the content-based distinction strategy is not concerned with the first difficulty pointed out against state-based distinction strategies: it seems compatible with the way in which types are formed through the rules of type-formation. Furthermore, to the extent that type-levels would depend only on the content of knowledge and not on some feature of the state, the problem of generality of the reading of K addressed to state-based distinction strategies does not affect a distinction of knowledge based on content. However, the fact that the content-based distinction strategy is immune to the problems of the state-based one is still not a reason for its adoption. For this further step, a difference in some actual feature of the content of knowledge able to reflect the distinction of types should be found.

A plausible suggestion in this direction seems to be to distinguish types on the basis of the distinction between knowledge of non-epistemic propositions – propositions about mere facts – and knowledge of epistemic propositions – propositions about knowledge. For example, the proposition “Mary knows that the sky is blue” holds an epistemic content, whereas the proposition “the sky is blue” does not. This distinction seems not to be artificial, but to reflect a real feature of our knowledge of such propositions. In fact, one could argue that knowledge of epistemic propositions involves some sort of self-reflection and introspection that knowledge of non-epistemic propositions lacks. This distinction apparently reflects the distinction of types because it is based on the epistemic content of the known proposition. Thus, the former distinction seems able to provide justification for the latter.

However, even if this analogy seems able to justify the introduction of types, there are some reasons for doubting its effectiveness. First, the distinction between propositions holding epistemic and non-epistemic content is not as clear as it looks at first sight. Obtaining some non-epistemic state of affairs concerning agents could also have effects on the agents’ epistemic state. “John is lying on the bed” is not an epistemic proposition. It does not involve any occurrence of the verb “to know”, but it does prevent John from knowing what is happening in the kitchen now. Thus, it can be credited with having some epistemic import. Conversely, every agent’s change of epistemic conditions also affects his psycho-physical state and has consequences for the agent’s body and for the environment, which can be considered factual matters. It is also a mistake to ground the distinction between epistemic and non-epistemic propositions on the supposition that knowledge of epistemic propositions requires some sort of self-reflection and introspection that other propositions lack. One could conclude that he does not know what is happening in the kitchen now without the help of any introspection, simply by considering the fact that he is now lying on the bed and then inferring that he does not know what
is currently happening in the kitchen. Therefore, we can conclude that there is not a clear criterion of distinction between propositions about facts and propositions about epistemic states; propositions of the first kind are strictly connected to propositions of the second, and *vice versa*, and there are borderline cases in which it is not clear whether a proposition is epistemic or not. This would make it problematic for such a distinction to reflect the neat hierarchy of type-levels.

Another reason for doubting the capacity of such a distinction to reflect types is that it is a questionable matter whether complex propositions such as the problematic proposition (2) \((p \land \neg Kp)\) in the paradox can be considered epistemic. Proposition (2) is about the ignorance of a certain fact. Intuitively, it is not a proposition about the epistemic state of an agent. But for the effectiveness of the solution, proposition (2) must be an epistemic proposition. Otherwise, if proposition (2) were non-epistemic, it would be of type 0; consequently, knowledge of this proposition would not be, as in (5*), of type 2, but of type 1, and the paradoxical conclusion would not be avoided.

A further reason for arguing that such a distinction is unable to justify the typed account of knowledge is that it does not reflect the distinction between type-levels, and so cannot be identified with it. In fact, the type-distinction does not allow only a distinction between zero-level types corresponding to non-epistemic propositions and first-level types corresponding to epistemic propositions. There can be higher-than-first-level types. This fact is not of secondary importance, but a necessary condition for the effectiveness of the type-strategy against Fitch’s argument. In fact, suppose an instance of the *Knowability Paradox* in which proposition \(p\) in (5) is of a higher-than-first-level type. In this case, all the propositions involved in the argument would be epistemic, even if they were to have different type-levels. For this reason, the distinction between epistemic and non-epistemic content of propositions cannot be an adequate correlative of the type-distinction, and thus it cannot motivate the latter.

Are there other not-principled ways to distinguish types through a distinction of knowledge contents? We can exclude distinctions not based on some epistemic component of the content because, if type-levels of \(K\) were not determined by type-levels of other occurrences of knowledge in the content of \(K\), then the hierarchy of knowledge-types necessary for the solution of the paradox would not be granted. Let us illustrate the problem. Take proposition (5) \(K(p \land \neg Kp)\). The type-level of the occurrence of \(K\) outside the brackets should be higher than the type-level of \(K\) inside the brackets; otherwise, the former type-level would collapse into the latter. If there is such a collapse, the paradox is still in place. Thus, a type-level hierarchy effective against the paradox requires a definition of types preventing the collapse. But the only way to grant the hierarchy is by defining the type-level of the occurrence of \(K\) outside the brackets as being higher than the type-level of \(K\) inside the brackets, i.e., by defining type-levels by reference to the
type-level of other occurrences of K within the scope of K itself. Now, if the type-levels of an occurrence of K must be determined by the type-level of occurrences of K in its content, the reference to some aspect of the epistemic content in the determination of types is required. But we do not see any unprincipled way to maintain a distinction in content based on its epistemic features being at the same time intuitively imputable to the ordinary concept of knowledge and able to reflect the complexity of the type-hierarchy.

However, we advance a further argument against content-based distinction strategies. This argument is valid in general for every content-based distinction strategy. Take, for example, the account presented in section 3. According to the rules of type-formation, the type-level of each occurrence of K is determined by the highest type-level of other occurrences of K in the content of K itself. In particular, the type of K is equivalent to the type of the proposition under its scope having the highest type-level plus one. For example, the K-operator applied to proposition \((2*)\), \(p_0 \land \neg K_1 p_0\), is level two, because the proposition in \((2*)\) holding the highest type-level is \(\neg K_1 p_0\), a proposition of level one. From this, it correctly follows \((5*)\), \(K_2 (p_0 \land \neg K_1 p_0)\).

But what happens when one applies the rule of distributivity over conjuncts to \((5*)\)? For the solution to be effective, it should follow, as stated in section 3, that:

\[(6*) K_2 p_0 \land K_2 \neg K_1 p_0\]

Then, by factivity:

\[(7*) K_2 p_0 \land \neg K_1 p_0\]

\((7*)\) is not contradictory because the two occurrences of K in \((7*)\) hold different types. Thus, as the defender of the type-solution argues, the paradox would be blocked at this step. However, this is not the case. In fact, given \((5*)\), by distributing K over the two conjuncts, the type-level of the K-operator applied to each conjunct will depend, by definition, on the type-level of the proposition under its scope. The result will be:

\[(6**) K_1 p_0 \land K_2 \neg K_1 p_0\]

In \((6**)\) the K-operator applied to the first conjunct is not level two, as the defender of the type-solution argues, but level one. This is because its type-level is determined by the type-level of the proposition under its scope plus one. The level of p is zero, and zero plus one is one. From \((6**)\) and factivity we obtain:

\[(7**) K_1 p_0 \land \neg K_1 p_0\]

and \((7**)\) is a contradiction. The argument is not blocked at this step and the paradoxical conclusion is met again. Therefore, the suggested type-solution is unable to resolve the Knowability Paradox. Notice that the responsibility for this

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failure is the distinction of types based on the distinction of knowledge contents. If types are defined by the type-level of their content, changing content changes type-level to the extent in which content has a different type-level. The distribution of K over the two conjuncts in (6**) results in two occurrences of K with contents having different type-levels with respect to the type-level of proposition (2); consequently, there is a difference between the type-level of those occurrences and the type-level of K in (5**). Notice that the argument does not depend in any way on the specific formulation given in section 3; it is proper for every type-level distinction of occurrences of K based on a distinction of the type-level of their content. It can be generalized to every content-based distinction strategy.22

7. Conclusion

In sections 5 and 6 we argued that neither a distinction in some features of states of knowledge nor a distinction in features of content of knowledge can reflect the distinction of type-levels of knowledge required to solve the Knowability Paradox. We argued that these two possible strategies exhaust the possible ways in which types can be grounded in real features of knowledge. A consequence is that types cannot reflect any actual feature possessed by knowledge. Thus, if one abstracts from other possible independent considerations supporting reasons for typing knowledge and considers the motivation for a type approach with respect to the Knowability Paradox alone, there are no substantive reasons to independently motivate the type-introduction except the fact that the type-distinction escapes the paradox. Therefore, the introduction of knowledge types in the specific case of the Knowability Paradox is ad hoc.

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22 The question remains whether there is some way out of this impasse. The only possible suggestion that comes to mind is to introduce the following restriction on the distributivity of K over conjunction: distributivity is allowed only if all the conjuncts in the conjunction hold the same type-level. In this way, the step from (5*) to (6*) is not allowed, and the paradox is blocked. However, this move is problematic because it places a restriction on a property of knowledge, distributivity, which seems to be almost universally accepted and unquestionable. Thus, it misrepresents knowledge and, consequently, is completely ad hoc.
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